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Kinetic Description of a Degenerate, Rotating, Non-neutral Electron Plasma in External Magnetic Fields in the Framework of the Thomas-Fermi-Dirac Theory

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Abstract. Aim of this work is to extend the results obtained in a previous study on the magnetic confinement and stability of a quantum degenerate non-neutral fermion plasma. This extension consists in the inclusion in the previously set up model of the effects of the exchange forces, and generalises the Thomas-Fermi (TF) approach used in the referenced work towards a Thomas-Fermi-Dirac (TFD) statistical description. The TF model has not only been used extensively and with success in these years to study atomic, nuclear and molecular properties, or to evaluate features of matter in extreme conditions such as low temperatures and/or high densities typical of astrophysics and inertial confinement fusion experiments, but also to found hydrodynamic theories for the diffusion and stability of fermion plasmas, one component non-neutral degenerate fluids, plasmas etc. In this paper an equation for density profiles in cylindrical symmetry is found, from the semiclassical kinetic theory of quantum gases, which takes into account the effects of temperature, average velocity, external magnetic field and quantum exchange. Numerical solutions of this equation for the case of complete quantum degeneracy are given and comparisons with the previous results are carried out.

INTRODUCTION

A many-body charged particle system in which there is not overall charge neutrality can have, as has been shown since the early '70s, collective oscillations and excitations as well as shielding and screening effects [1]; this many-body collection of charged particles is therefore termed a non-neutral plasma (NNP). In certain physical situations the NNP is in such conditions that also global effects of quantum nature are present and observable [2,3,4]. These conditions, which append the adjective "degenerate" to an NNP, are realized when the average distance between neighboring particles is of the order of the dimension of the quantum wave field of the particles themselves [5] (de Broglie length); this in turn depends on the number density and temperature of the plasma itself: the lower the temperature and the higher the density, the higher quantum effects are and more and more degenerate the NNP is. Shielding is

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highly affected by quantum effects and the screening of the potentials, with its influence on the spatial dependence of density, is a main subject of this paper. In this work, in fact, the results of a previous study [6] on the magnetic confinement and stability of a quantum degenerate fermion NNP in the presence of external forces are extended by including the role of exchange potentials on the particles; the extension consists then in taking the description from a Thomas-Fermi level to a Thomas-Fermi-Dirac one. Density profiles in cylindrical coordinates and symmetry will be computed for a completely degenerate electron NNP. Semiclassical kinetic theory of gases, i.e. the Boltzmann-Uehling-Uhlenbeck (BUU) equation, and quantum statistics in the form of the Fermi statistics are the starting point for this analysis.

THE MODEL

For the case of an electron gas the BUU equation for the electron distribution function f can be written [4,6] as

$$\frac{\partial f}{\partial t}+\mathbf{v}\cdot\frac{\partial f}{\partial \mathbf{r}}+\frac{\mathbf{F}}{m}\cdot\frac{\partial f}{\partial \mathbf{v}}=\int\left(f'f'_1(1-\gamma f)(1-\gamma f_1)-ff_1(1-\gamma f')(1-\gamma f'_1)\right)g\sigma(g,\chi)\,d\Omega\,d\mathbf{v}_1 \tag{1}$$

where $\gamma=\dfrac{h^3}{2m^3}$. Since a stationary solution with density vanishing at infinity is expected, the form of f is known to be a local Fermi distribution [7], for which the collision term of eq. (1) is vanishing; it is more convenient then to make use of the H theorem and consequently write the equation in the structure adequate for the logarithm of the distribution function [7]; if a finite average velocity $\mathbf{v}_0$ must also be taken into account, then eq. (1) reduces to:

$$(\mathbf{v}_0+\mathbf{c})\cdot\frac{\partial \ln f}{\partial \mathbf{r}}+\left(\frac{\mathbf{F}}{m}-\left(\mathbf{v}_0\cdot\frac{\partial}{\partial \mathbf{r}}\right)\mathbf{v}_0\right)\cdot\frac{\partial \ln f}{\partial \mathbf{c}}-\frac{\partial \ln f}{\partial \mathbf{c}}\mathbf{c}:\frac{\partial \mathbf{v}_0}{\partial \mathbf{r}}=0 \tag{2}$$

where $\mathbf{c}$ is the peculiar velocity. Introducing, the local Fermi distribution function

$$f_F(\mathbf{r},\mathbf{c})=\frac{\gamma^{-1}}{A(\mathbf{r})e^{\frac{mc^2}{2kT(\mathbf{r})}}+1}=\frac{\gamma^{-1}}{e^{\frac{\frac{1}{2}mc^2-\mu(\mathbf{r})}{kT(\mathbf{r})}}+1} \tag{3}$$

into eq. (2) the following equation is obtained:

$$\frac{e^{\frac{mc^2}{2kT}}}{e^{\frac{mc^2}{2kT}}+1}\left[(\mathbf{v}_0+\mathbf{c})\cdot\left(\frac{\partial A}{\partial \mathbf{r}}-A\frac{mc^2}{2kT^2}\frac{\partial T}{\partial \mathbf{r}}\right)+\left(\frac{\mathbf{F}}{m}-\left(\mathbf{v}_0\cdot\frac{\partial}{\partial \mathbf{r}}\right)\mathbf{v}_0\right)\cdot\mathbf{c}\frac{mA}{kT}-\frac{mA}{kT}\mathbf{cc}:\frac{\partial \mathbf{v}_0}{\partial \mathbf{r}}\right]=0 \tag{4}$$

If eq. (4) is to be satisfied then all the coefficients of successive powers of the peculiar velocity between square brackets must identically vanish; hence:

$$\frac{mA}{kT}\mathbf{cc}:\frac{\partial \mathbf{v}_0}{\partial \mathbf{r}}=0, \quad \mathbf{v}_0\cdot\frac{\partial A}{\partial \mathbf{r}}=0 \;\Rightarrow\; \mathbf{v}_0\perp\frac{\partial A}{\partial \mathbf{r}} \tag{5a,b}$$

$$A\frac{mc^2}{2kT^2}\mathbf{c}\cdot\frac{\partial T}{\partial \mathbf{r}}=0 \;\Rightarrow\; \frac{\partial T}{\partial \mathbf{r}}=0, \quad \frac{\partial A}{\partial \mathbf{r}}+\frac{A}{kT}\left(\mathbf{F}-m\left(\mathbf{v}_0\cdot\frac{\partial}{\partial \mathbf{r}}\right)\mathbf{v}_0\right)=0 \tag{5c,d}$$

These equations are as general as eq. (4) but are indeed extremely useful; in particular eq. (5c) states that the temperature must be constant and eq. (5d) establishes an equilibrium relation between the various external and internal forces acting over the gas. It also provides a relation between the chemical potential μ and all other forces.

We consider now an electron gas, in condition of cylindrical symmetry with respect to $\mathbf{z}$ axis, described through cylindrical coordinates $\mathbf{r}=[r,\vartheta,z]$. The external force is given by a constant and uniform magnetic field along the symmetry axis, $\mathbf{B}=B\mathbf{z}$; the remaining internal force is given by the self-consistent Vlasov electric field. Under these conditions all the quantities in eq. (5d) will depend at most on r alone. This allows a simple form of $\mathbf{v}_0$ to be found through eq. (5a); taking into account that to have stable equilibrium no radial velocity must be present, we have that $\mathbf{v}_0=v_{0z}\mathbf{z}+\boldsymbol{\omega}\times\mathbf{r}$, where $\boldsymbol{\omega}=\omega\mathbf{z}$ is a constant vector, directed along the symmetry axis, representing an angular velocity for the gas. Now, inserting this expression and the Lorentz force into eq. (5d), only a scalar equilibrium equation that connects the gradient of the chemical potential, the Lorentz force and the centrifugal force remains:

$$kT\frac{d}{dr}(\ln A)-e(E_r+\omega rB)+m_e\omega^2 r=0 \tag{6}$$

Expressing the electric field as $E_r=-\frac{dV}{dr}$, with as b.c. $\lim_{r\to 0^+}V(r)=0$, we get:

$$\mu(r,T)=\mu(0,T)+eV(r)+\frac{1}{2}m\omega(\omega-\omega_c)r^2 \tag{7}$$

where $\mu(0,T)$ is the chemical potential for an uniform and homogeneous electron gas [4,5,8,9,10], i.e. the value of the chemical potential when no forces are present, and $\omega_c=\frac{eB}{m_e}$ is the cyclotron frequency, representative of the external magnetic field. The electric potential exchange effects, of purely quantum nature, are taken into account, in the simplest way, through the Dirac and Slater formalism [8,9,10,11,12], by adding to eq. (7) a suitable term proportional to the cubic root of the local particle density (local density approximation, LDA), as happens in the usual Thomas-Fermi-Dirac, TFD, theory of atoms. This means that it is assumed here an equal number of upward- and downward- oriented spins. In this way the repulsive effects between electrons of parallel spins, generating the s.c. "Fermi hole" or "exchange hole" around a generic test particle, are considered; however, attractive effects due to other kinds of correlations (f.i. attraction between antiparallel spins) are not included. Of the many relativistic or non-relativistic LDA models [13] the non-relativistic Dirac-Slater, DS, one has been chosen because of its simplicity, both from a mathematical point of view and from the physical one. The fact that it is not a relativistic potential is coherent with the other assumptions of the present model. In addition, as it happens with other exchange potentials, the DS one can easily be related to variational forms and inserted in the larger theoretical frame of density functional methods. In fact the DS potential can be derived from the following density functional:

$$E_X^D[n] = -C_X \int n^{4/3}(\mathbf{r})d\mathbf{r} \tag{8}$$

where, in SI units, $C_X = \frac{3}{4}\frac{e^2}{4\pi\varepsilon_0}\left(\frac{3}{8\pi}\right)^{1/3}$. So, the DS exchange potential is simply:

$$W_X^D = -\frac{e^2}{4\pi\varepsilon_0}\left(\frac{3n(\mathbf{r})}{8\pi}\right)^{1/3}. \tag{9}$$

It must be observed here that the rhs of eq. (9) cannot be added directly to eq. (7), as is usually done in the TFD method; in fact, in the present model, eq. (7) is naturally linked to the condition, different from the usual TFD one and deriving from the above stated b.c. for $V(r)$, that for $r \to 0^+$ the fermions must posses the characteristics of an uniform and homogeneous gas; because $n(0) \neq 0$ then a very important normalizing term must also be added, so that eq. (7) finally becomes:

$$\mu(r,T) = \mu(0,T) + eV(r) + \frac{1}{2}m\omega(\omega-\omega_c)r^2 - \frac{e^2}{4\pi\varepsilon_0}\left(\frac{3n(r)}{8\pi}\right)^{1/3} + \frac{e^2}{4\pi\varepsilon_0}\left(\frac{3n(0)}{8\pi}\right)^{1/3} \tag{10}$$

Inserting eq. (10) into the Fermi distribution function and integrating over all the velocity space via Sommerfeld's Lemma at first order, one gets the following relation, generalized to take into account finite temperature and average velocity:

$$n(r) = \frac{8\pi}{3h^3}\left[2m_e\left(eV(r) + \frac{1}{2}m_e\omega(\omega-\omega_c)r^2 + \mu_0 - \frac{e^2}{4\pi\varepsilon_0}\left(\frac{3(n(r)-n(0))}{8\pi}\right)^{1/3}\right)\right]^{3/2} \cdot \left[1 + \frac{(\pi kT)^2}{8}\left(eV(r) + \frac{1}{2}m_e\omega(\omega-\omega_c)r^2 + \mu_0 - \frac{e^2}{4\pi\varepsilon_0}\left(\frac{3(n(r)-n(0))}{8\pi}\right)^{1/3}\right)^{-2}\right] \tag{11}$$

where $\mu_0 = \mu(0,0) \equiv E_F$ is the s.c. Fermi energy.

For the case of complete degeneracy ($T=0$) one gets from eq. (11):

$$an^{2/3} + br^2 + c + dn^{1/3} - dn_0^{1/3} = eV \tag{12}$$

where $n(0) \equiv n_0$, $a = \frac{1}{2m}\left(\frac{3h^3}{8\pi}\right)^{2/3} > 0$, $b = -\frac{1}{2}m\omega(\omega - \omega_c)$, $c = -\mu_0 < 0$,

$d = \frac{e^2}{4\pi\varepsilon_0}\left(\frac{3}{8\pi}\right)^{1/3} > 0$. For $r \to 0^+$ eq. (12) tends to $an_0^{2/3} + c = 0$ which is the usual expression for an uniform and homogeneous electron gas. Also, for $d \to 0^+$ eq. (12) gives the simple TF equation in absence of exchange forces. Requiring now the self-consistency of electron density and electric potential, imposing that eq. (12) satisfy Poisson's equation, a non-linear equation for only the electron density remains:

$$n'' - \frac{1}{3n}(n')^2 + \frac{n'}{r} = \frac{n^{2/3}}{\left(2/3\, an^{1/3} + 1/3\, d\right)}(gn - 4b) \tag{13}$$

where $g = \frac{e^2}{\varepsilon_0} > 0$, with particle number normalization and cylindrical symmetry b.c.:

$$\lim_{r \to 0^+} n(r) = n(0) = n_0, \quad \lim_{r \to 0^+} \frac{dn}{dr} = 0. \tag{14}$$

ANALYTICAL RESULTS

A solution vanishing at infinity for eq. (13) is obtainable if and only if [6] its rhs near the origin is negative; this means that to have confinement it must be $gn_0 < 4b$, *independently* of the value of *d*. This important result shows that exchange effects don't alter at all the s.c. confinement region [6], hence they don't alter the value of the confining necessary magnetic field too, for the considered gas at the given angular velocity. Exchange effects do indeed alter the confinement radius for the gas, in the direction of extending it above that of the purely TF model, as is to be expected, as stated above, because of the presence of "Fermi holes".

NUMERICAL RESULTS, COMPARISONS AND DISCUSSION

Eq. (13) has been solved numerically. Figure 1 compares, at the same number of confined particles per unit length, magnetic field and angular velocity, in doubly-logarithmic scale, the TF and TFD density profiles. The role of the repulsive exchange holes is distributed between the two effects of lowering the density at the origin of the TFD solution and of increasing its confinement radius.

CONCLUSIONS

The quantum exchange effects between parallel-spinned electrons have been included in a semiclassical model of a one-component, non-neutral, degenerate, fermion plasma. The plasma rotates around its simmetry axis in a cylindrical configuration and is magnetically confined. In this way the model has been extended

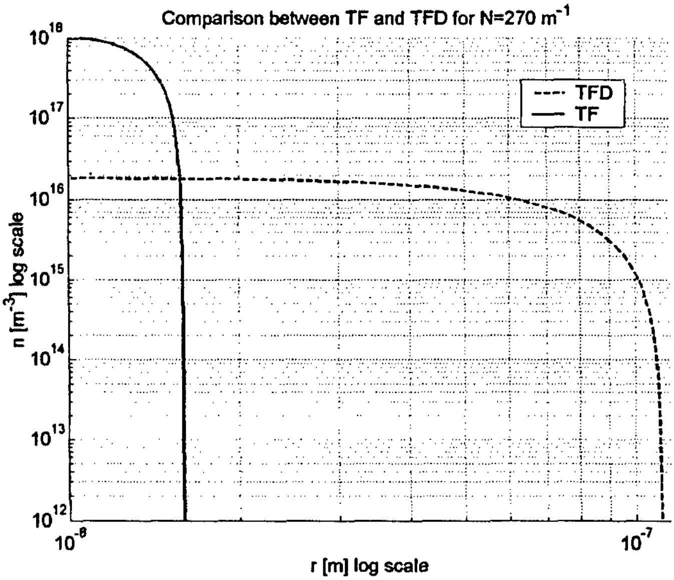

FIGURE 1. Comparison between TF and TFD models for N=270 m^{-1}, ω = 4.2E+6 Hz and ω_c = 8.0E+14 Hz.

through a local density approximation from a TF description level to a TFD one, as it happens f.i. for the electronic theory of atoms. The usual expression of the exchange potential has been modified for the special needs of the model. The equation for the density profile of the plasma has been solved numerically. It is seen that the s.c. confinement region is not altered by the exchange effects, even if the confinement radius becomes larger because of the repulsive effects of the "Fermi holes".

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